Correlation and Coherence between India and Global Equity Market in Time Frequency Localization Through Wavelet Transformation

Aasif Shah¹, Malabika Deo² and Wayne King³

Abstract

Given the heterogeneous trading behaviour in stock markets, investors operate at different frequencies for their trade and investment preferences. Viewing the phenomenon through portfolio diversification context, this implies that short term investors are interested in stock returns at higher frequencies, that is, short-term fluctuations and the long term investors are interested at lower frequencies, that is, long-term fluctuations. Using pair-wise wavelet correlation and cross-correlation, we examine the implications of Indian stock market integration with rest of the global equity markets for diverse investment horizons which is not possible to figure out otherwise with conventional time domain analysis. Our results show that Indian stock market is highly integrated with global equity markets at lower frequencies and comparatively less integrated at higher frequencies. The cross correlation results also confirm that the integration of Indian market becomes stronger as we move to lower frequency scales. The results from wavelet coherence suggest that this correlation becomes strong during crises period at lower frequency resolutions. Overall the findings are plausible and have strong policy implications.

JEL Classifications: F36, G11, G15

Key words: Global Equity Markets, Wavelet Correlation & Cross Correlation, Wavelet Coherence

1. Introduction

With the development in the liberalization of capital movements and the securitization of stock markets, international financial markets have become increasingly interdependent. The level of interaction or interdependence between markets has important consequences in terms of predictability, portfolio diversification and asset allocation. Theory predicts that gains can be achieved through international portfolio diversification if returns in various markets are not perfectly correlated. Policies of deregulation and the liberalisation of capital markets, coupled with technological advances suggest that markets have become more integrated over time and hence opportunities for portfolio diversification are reduced. The study of stock market integration is very crucial in finance given the consequences for asset allocation decisions and portfolio diversification. The concept has received the long history of theoretical and empirical investigations. For example; Ever since Grubel (1968), the opening treatise on the benefits of international portfolio diversification, issues related to co-movement of stock market returns have received lot of attention in international finance. Since then, a plethora of research activity has emerged on the co-movement of international stock prices (see, for example, Lin et al. (1994), Karolyi and Stulz (1996), Forbes and Rigobon (2002), Brooks and Del Negro (2004), Yang. J et al (2006)). In the early time periods, the study of stock market linkages had been undertaken through simple correlation coefficient (see, for example, Brooks and Del Negro (2004), Debjiban Mukherjee (2007)).
However, off late, more refined methods like rolling window correlation (see, for example, Brooks and Del Negro (2004)), non-overlapping sample periods (see, for example, King and Wadhwani (1990) and Lin et al. (1994)) and cointegration (See, for example Voronkova, S. (2004)) have also been put into use.

In Asia, studies based on Co-integration have investigated the extent to which stock markets in the region are integrated and, in turn, have some implications to diversification opportunities in Asian stock markets (Chan, Gup, and Pan, 1992; Hung and Cheung, 1995; DeFusco, Geppert, and Tsetsekos, 1996; Masih and Masih, 2001). Other related studies like Chung and Liu, 1994; Arshanapalli, Doukas, and Lang, 1995; Cheung, 1997; Janakiramanan and Lamba, 1998; Dekker, Sen, and Young, 2001 use vector autoregression (VAR) techniques, including co-integration, Granger causality, impulse response analysis, and forecast error variance decomposition. In general, these studies offer mixed empirical evidence with respect to both long-run and short run relationships. Models based on co-integration and Error correction however are plagued with several issues. For example, these models have been designed to deal with just two time frames or frequencies. In the Indian context Mukherjee and Bose (2006), Mukhopadhyay (2009), Mukherjee and Mishra (2010) and Siddiqui and Seth (2010) employed tools like correlation, cross correlation, Granger causality test and Garch models to study inter-linkages between the Indian stock market and some other emerging and developed markets. However, given the heterogeneous trading behaviour in stock markets, participants in these markets operate at different frequencies. Viewing through the portfolio diversification context, this means short term investors are interested in stock returns at higher frequencies, that is, short-term fluctuations, medium term investors at medium frequencies and the long term investors are interested in the relationship at lower frequencies, that is, long-term fluctuations. Therefore it is important to assess the stock market co-movement at more than two frequencies. The wavelet approach allows the study of the frequency components and time information in time series, in contrast with the standard time series econometric models which consider only one or at most two time scales (the short and the long run) and rely on model parameters. Recent application of wavelet analysis either use pair wise wavelet correlation or regression within the set of multiple stock indices while examining the return spill over effects between stock e.g. Lee (2004), Sharkasi et al. (2004), Fernandez (2005), Rua and Nones (2009) and Raghavan et al (2010). Recently Aviral Tiwari, et al., (2013) used wavelet multiple correlation and found the Asian Markets are integrated strongly at lower frequencies.

Using wavelet multi-resolution analysis to examine the stock market integration of a particular market with Global markets has not been experimented so far. Nor do the studies have incorporated the applications of wavelet coherence to overcome the issues plagued in wavelet correlation transform. We contribute to the literature from both ends that is by studying the market integration comprehensively through wavelet correlations as well through wavelet coherence (power spectrum) by focusing on the time and frequency localization to provide additional insights in dynamic linkages of Indian equity market with rest of global equity markets. The results penetrated are the unique contribution of the study.

The remainder of the paper is organised as follows. Section 2 describes the Multi-scale decomposition methodology. Section 3 illustrates the wavelet correlation. Section 4 discusses about the wavelet cross covariance & correlation. Section 5 discusses about wavelet coherence. Section 6 gives a frame work of the data description and discussion of results and finally, Section 7 draws the conclusions and policy implications.

The classic time domain approach aims at studying the underlying properties of an economic variable whose realizations are recorded at a predetermined frequency. This approach does not convey any information regarding the frequency components of a variable. Thus it makes the pretended assumption that the relevant frequency to study the behaviour of the variable matches with its sampling frequency. However, an issue arises if the variable realizations depend on a much complicated manner on several frequency components. Wavelet techniques possess an inherent ability to decompose the time series into several sub-series which may be associated with a particular time scale. In particular, wavelet methods present a lens to the researcher, which can be used to zoom on the details and draw an overall picture of a time series in the same time. In a way one could say that with wavelet methods we are able to see both the forest and the trees.

Until recent years the stock market integration has been mostly analyzed following conventional time-domain approach where the frequency domain had been ignored. However a quite possibility is there that the relationship would differ at different frequencies due to heterogeneous trading nature of market participants. The dynamic linkage between stock markets can therefore vary across frequencies and such relationship may even change over time. Given its ability to decompose the time series data into different time scales and modelling financial market heterogeneity, wavelet approach is pertinent. The wavelet and scaling coefficients at the first level of decomposition are obtained by convolution of the data series with the father wavelet \( \phi(t) \) and mother wavelet \( \psi(t) \)

\[
\int \psi(t) dt = 0, \quad \int \phi(t) dt = 1
\]

(1)

The father wavelets are used for the low frequency smooth components parts of a signal and the mother wavelets are used for the high-frequency details components. That is father wavelets are used for the trend components and mother wavelets are used for all the deviation from trend. Hence, a sequence of mother wavelets is used to represent a function and only one father wavelet is used to represents a function. To continue the frequency- by-frequency decomposition of the original signal, one typically resorts to what is known as the pyramid algorithm see (plot 1). In short, wavelet literature has emerged with a number of wavelets families. However, in the empirical literature the majority

of the literature is concentrated to the use of orthogonal wavelets such as the Haar, Daubelets, Symmlets and coiflets. A time series, say \( f(t) \), can be decomposed by the wavelet transformation, which can be expressed as follows:

\[
f(t) = \sum_{j,k} s_{j,k} \phi_{j,k}(t) + \sum_{j,k} d_{j,k} \psi_{j,k}(t) + \sum_{j-1,k} d_{j-1,k} \psi_{j-1,k}(t) + \ldots + \sum_{1,k} d_{1,k} \psi_{1,k}(t)
\]

(2)

\(^4\) Frequency is the rate of change with respect to time. Change in a short span of time means high frequency. Change over long span of time means low frequency. If a signal does not change at all, its frequency is zero. If a signal changes instantaneously, its frequency is infinite.
where \( J \) is the number of multiresolution levels, and \( k \) ranges from 1 to the number of coefficients in each level. The wavelet coefficients \( s_{j,k}, d_{j,k}, \ldots \), \( d_j \) are the wavelet transform coefficients and \( \phi_{j,k}(t) \) and \( \psi_{j,k}(t) \) represents the approximating wavelets functions. The wavelets transformations can be expressed as

\[
\begin{align*}
 s_{j,k} &= \int \phi_{j,k}(t) f(t)dt \quad (3) \\
 d_{j,k} &= \int \psi_{j,k}(t) f(t)dt, \text{ for } j=1,2,\ldots,J. \quad (4)
\end{align*}
\]

where \( J \) is the maximum integer such that \( 2^J \) takes value less than the number of observations.

The detail coefficients, \( d_{j,k}, \ldots, d_{j,k} \), represents increasing finer scale deviation from the smooth trend and \( s_{j,k} \) which represent the smooth coefficient capture the trend. Hence, the wavelet series approximation of the original series \( f(t) \) can be expressed follows:

\[
f(t) = S_{j,k}(t) + D_{j,k}(t) + D_{j-1,k}(t) + \ldots + D_1(t). \tag{5}
\]

where \( S_{j,k} \) is the smooth signal and \( D_{j,k}, D_{j-1,k}, D_{j-2,k}, \ldots, D_{1,k} \) detailed signals. These smooth and detailed signals are expressed as follows:

\[
S_{j,k} = \sum_k s_{j,k} \phi_{j,k}(t), \quad D_{j,k} = \sum_k d_{j,k} \psi_{j,k}(t), \quad \text{and} \quad D_{j,k} = \sum_k d_{j,k} \psi_{j,k}(t). \tag{6}
\]

The \( S_{j,k}, D_{j,k}, D_{j-1,k}, D_{j-2,k}, \ldots, D_{1,k} \) are listed in increasing order of the finer scale components.

3. **Wavelet Correlation**

The estimation of wavelet correlation involves the construction of variances of \( \{x_t, y_t\} \) and covariances \( \{x_t\} \) and \( \{y_t\} \) at different wavelet scales. Wavelet variance essentially refers to the substitution of variability over certain scales for the global measure of variability estimated by sample variance. The wavelet variance of stochastic process \( X \) is estimated using the MODWT coefficients for scale \( \tau_j = 2^{j-1} \) through:

\[
\hat{\sigma}_x^2(\tau_j) = \frac{1}{N_j} \sum_{k=L_{j-1}}^{N-1} (\hat{W}_{j,k})^2
\]

Where \( \hat{W}_{j,k} \) the MODWT wavelet coefficient of variable \( X \) at scale is \( \tau_j \). \( \hat{N}_j = N = L_j + 1 \) is the number of coefficients unaffected by boundary, and \( L_j = (2^{j-1})(L-1) + 1 \) is the length of the scale \( \tau_j \) wavelet filter.

The wavelet covariance decomposes the covariance between two stochastic processes on a scale-by-scale. The wavelet covariance at scale \( \tau_j \) can be written as follows:

\[
\gamma_{xy}(\tau_j) = \text{cov}_{xy}(\tau_j) = \frac{1}{N_j} \sum_{k=L_{j-1}}^{N-1} \hat{W}_{j,k}^x \hat{W}_{j,k}^y
\]

Given the wavelet covariance for \( \{x_t, y_t\} \) and wavelet variances for \( \{x_t\} \) and \( \{y_t\} \), the MODWT estimator of wavelet correlation can be expressed as follows:

\[
\hat{\rho}_{xy}(\tau_j) = \frac{\text{Cov}_{xy}(\tau_j)}{\hat{\sigma}_x(\tau_j) \hat{\sigma}_y(\tau_j)}
\]

4. **Wavelet Cross Covariance and Correlation**
The cross correlation is a more powerful tool for examining the relationship between two time series. The cross correlation function considers the two series not only simultaneously (at lag 0), but also with a time shift. The cross correlation reveals causal relationships and information flow structures in the sense of Granger causality. If two time series were generated on the basis of a synchronous information flow, they would have a symmetric lagged correlation function, \( \rho_{\tau} = \rho - \tau \) the symmetry would be violated only by insignificantly small, purely stochastic deviations. As soon as the deviations between \( \rho \) and \( \rho - \tau \) become significant, there is asymmetry in the information flow and a causal relationship that requires an explanation. The cross correlation can be constructed utilizing the wavelet cross covariance. It is straightforward to derive the cross covariance, once the wavelet covariance is derived. For \( N \geq L_j \) a biased estimator of the wavelet cross covariance based on the MODWT is given by

\[
R_{XY, \tau} = \left\{ \begin{array}{ll}
\frac{1}{N} \sum_{T=L_j-1}^{N-\tau-1} \sum_{d} X_{j,d} Y_{j,d+\tau} & \text{for } \tau = 0, \ldots, L_j - 1 \\
\frac{1}{N} \sum_{T=L_j-1}^{N-\tau-1} \sum_{d} X_{j,d} Y_{j,d+\tau} & \text{for } \tau = -1, \ldots, -(L_j - 1) \\
0 & \text{Otherwise}
\end{array} \right.
\]

Allowing the two processes to differ by an integer lag \( \tau \), the wavelet cross correlation can be defined as

\[
\hat{\rho}_{s,k}(\tau_j) = \frac{\gamma_{s,k}(\tau_j)}{\hat{\sigma}_1(\tau_j)\hat{\sigma}_2(\tau_j)}
\]

The wavelet cross-correlation decomposes the cross-correlation between two time series on a scale-by-scale basis. Thus it becomes possible to see how the association between two time series changes with time horizons. Genaçay et al. (2002) define the wavelet cross-correlation as:

\[
\hat{\rho}_{s,k}(\tau_j) = \frac{\gamma_{s,k}(\tau_j)}{\hat{\sigma}_1(\tau_j)\hat{\sigma}_2(\tau_j)}
\]

where \( \hat{\sigma}_2(\tau_j) \) and \( \hat{\sigma}_2(\tau_j) \) is respectively the wavelet variances for \( x_{1,t} \) and \( x_{2,t} \) associated with scale \( \tau_j \), and the wavelet covariance between \( x_{1,t} \) and \( x_{2,t-k} \) associated with scale \( \tau_j \). The usual cross-correlation is used to determine lead-lag relationships between two time series; the wavelet cross-correlation gives a lead-lag relationship on a scale by scale basis.

**5. Wavelet Coherence (Power Spectrum)**

Cross wavelet power reveals areas with high common power. Another useful measure is how coherent the cross wavelet transform is in time frequency space. Following Torrence and Webster (1998) we define the wavelet coherence of two time series as

\[
R^{2}_p(s) = \frac{|S^{-1}(W_{XY}^s(s))|^2}{S^{-1}(|W_{X}^s(s)|^2) \cdot S^{-1}(|W_{Y}^s(s)|^2)},
\]

where \( S \) is a smoothing operator. Notice that this definition closely resembles that of a conventional correlation coefficient, and it is useful to think of the wavelet coherence as a localized correlation coefficient in time frequency space. We write the smoothing operator \( S \) as

\[ S(W) = S_{\text{scale}}(S_{\text{time}}(w_n(s))) \]
where S scale denotes smoothing along the wavelet scale axis and S time smoothing in time. It is natural to design the smoothing operator so that it has a similar footprint as the wavelet used. For the Morlet wavelet a suitable smoothing operator is given by Torrence and Webster (1998)

\[ S_{\text{integ}}(W)_s = \left( W_n(s) \ast c_1^s \right)_s, \]

\[ S_{\text{integ}}(W)_s = \left( W_n(s) \ast c_2 \Pi(0.6s) \right)_s, \]

Where c1 and c2 are normalization constants and 5 is the rectangle function. The factor of 0.6 is the empirically determined scale de-correlation length for the Morlet wavelet (Torrence and Compo, 1998). In practice both convolutions are done discretely and therefore the normalization coefficients are determined numerically.

6. Data, Results and Discussion

Our data set consists of weekly stock indices of thirty seven (37) equity markets among which 14 consists from Asia pacific, 11 from European, 09 from Middle East and 03 from American region between January, 01 1992 and June 30, 2013. However, the data for Middle East countries ranges from January, 01 2000 to June 30, 2013. The data was retrieved from the Bloomberg data base. In order to check the stationary of underlying variables, Augmented Dickey Fuller and Phillip Pheron test was carried out followed by basic descriptive statistics. Each return series were found to be stationary at first difference. The descriptive statistics revealed that most of stock markets are negatively skewed indicating that large negative stock returns are more common than large positive returns. While as Kurtosis statistics illustrated that all return series are leptokurtic with significantly fatter tails and higher peaks. The Jarque–Bera statistics for all the indices strongly reject the null hypothesis that their distributions are normal.

In order to estimate the wavelet correlation of Indian equity market with the rest of the Global Equity Markets, we begin by deconstructing the return series of indices in time frequency localization using Maximal Overlap Discrete Wavelet Transform (MODWT). Percival and Walden (2000), present five important properties which distinguish the MODWT from the discrete wavelet transform (DWT). (1) Although the DWT of level J restricts the sample size to an integer multiple of \(2^J\), the MODWT of level J is well defined for any sample size \(N\). (2) The MRA of the MODWT is associated with zero phase filters, implying that events, which feature in the original time series \(x_t\), may be poorly aligned with features in the MRA. (3) The MODWT details and smooth with zero phase filters make it possible to meaningfully line up features in an MRA with the original time series \(x_t\). (4) As is true for the DWT, the MODWT can be used to form an analysis of variance based on the wavelet and scaling coefficients. However, the MODWT wavelet variance estimator is asymptotically more efficient than the same estimator based on the DWT. (5) Where as a time series and a circular shift of the series can have different DWT-based empirical power spectra, the corresponding MODWT based spectra are the same. Given the sample of 1124 observations or roughly 22 years of data and 706 observation or roughly 14 years for Middle East data set, the maximum decomposition possibility is given by \(\lceil \log_2(T) \rceil\). However, for higher level decompositions, feasible wavelet coefficients get smaller. Therefore we choose to decompose the time series of stock returns into six details (\(w1\) to \(w6\)) and one (\(v6\)) smooth component. The wavelet scales are such that scale 1= 2-4 week period, scale 2= 4-8 week period, scale 3= 8-16 week period, scale 4=16-32 weeks period, scale 5=32-64 weeks period, scale 6= 64-128 weeks period and scale last represents smooth decomposition. Overall we generated 222 return

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5 A series is said to be stationary if the mean and auto-covariances of the series do not depend on time. To preserve the space, we have not provided results of Unit Root test and Descriptive Statistics. However, the estimation results are available on request.
series from 37 return series (37*6) without losing any data point or information. For several reasons, we do not report results for smooth series since it captures a long term fluctuation where predetermined frequency component is not known. After decomposing the given series into details and smooth’s, we proceed with wavelet correlation and cross correlation. The wavelet correlation between India and rest of the global markets with upper and lower bounds of 95 Percent confidence intervals are shown in Table 1 (Annexure 1). Overall the integration of Indian equity market with rest of global markets ranges from (15-55) degree over D1-D6 on an average. This implies that the investors with short term investment horizons can enjoy portfolio diversification and asset allocations. While on the other hand, least diversification opportunities are available for long term investors. Next we proceed with wavelet cross correlation function as a measure of similarity of two waveforms as a function of a time-lag applied to one of them. It is obtained similarly by allowing a certain 15 lags between observed and fitted values from the same linear combination at each of the wavelet scales. Figure 1 (Annexure 1) demonstrates as how the plotting of cross correlation between India and rest of the global markets were done. However, we report a single cross correlation plot in order to preserve the space. The results showed that most of the global markets lead Indian market at lower frequencies and the lead impact remain 3-4 months on an average. Given the time lead-lag structure, it is concluded that global equity markets could be predictable at lower frequencies rather than at higher frequencies.

It is worth to mention here that Wavelet correlation is plagued with several issues. For instance; if the study period is 2000-2014, the methodology would produce an average relationship between two markets over different time and frequency scales. However, an issues arises, when instead of an average, the market participants are interested to know this relationship in time frequency localization at each year so as to be able to identify the real relationship during crises and non crises period. Thus, keeping this vital concern into consideration, we make use of wavelet coherence (Power Spectrum) to overcome the limitations plagued with wavelet correlation. Our results Figure 2 (Annexure 1) demonstrate that the integration of Indian market with most of the global markets become strong during crises period. Interestingly, the evidence is supported from lower frequency components. This in turn means that portfolio diversification is possible even in crises period but during higher (short) frequency domains.

7. Conclusion and Policy Implications

The paper contributes to the literature by focusing on the simultaneous time and frequency intervals to provide additional insights in dynamic linkages of Indian equity market with rest of the global equity markets. In particular the study tried to establish the wavelet method as a new analytic technique to examine the scope of risk and portfolio diversification given the diverse investment horizons. The wavelet correlation shows that Indian market is correlated with the other global equity markets mainly on lower frequencies indicating that diversification opportunities are more likely obtainable at higher frequencies of returns. Further it is observed that the cross correlation between India and rest of the global markets become stronger as we move to lower frequency scales. It is concluded that global equity markets could be predictable at lower frequencies rather than at higher frequencies of specified data. Finally the results from wavelet coherence suggest that the integration of Indian market with most of the global markets become strong during crises period. Interestingly, the evidence is supported from lower frequency components. This in turn

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6 Again to preserve the space, we do not provide plots for wavelet cross correlation. However, we report one cross correlation plot between stock returns of India & Japan to help out the readers as how plotting estimation was carried out. Nevertheless, rest of the plots are available on request.
means that portfolio diversification is possible even in crises period but during higher (short) frequency domains. The findings are plausible and have strong policy implications.

References


APPENDEX-1

Table 1
### Wavelet Correlation of India with Asia Pacific Countries

<table>
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<tr>
<th>Countries</th>
<th>Pearson Correlation</th>
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<th>D3</th>
<th>D4</th>
<th>D5</th>
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### Wavelet Correlation of India with European Countries

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<tr>
<td>Ireland</td>
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<td>0.32</td>
<td>0.38</td>
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</tr>
<tr>
<td>Sweden</td>
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<td>0.36</td>
<td>0.5</td>
<td>0.45</td>
</tr>
<tr>
<td>Switzerland</td>
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<td>0.29</td>
<td>0.27</td>
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<td>0.32</td>
<td>0.45</td>
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</tr>
<tr>
<td>Spain</td>
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<td>0.33</td>
<td>0.35</td>
<td>0.55</td>
<td>0.65</td>
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</table>

### Wavelet Correlation of India with Middle East Countries

<table>
<thead>
<tr>
<th>Middle East</th>
<th>Pearson Correlation</th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>D4</th>
<th>D5</th>
<th>D6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Egypt</td>
<td>0.28</td>
<td>0.18</td>
<td>0.26</td>
<td>0.3</td>
<td>0.5</td>
<td>0.57</td>
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<td>Jordan</td>
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<td>0.17</td>
<td>0.15</td>
<td>0.19</td>
<td>0.47</td>
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<tr>
<td>Istanbul</td>
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<td>0.22</td>
<td>0.3</td>
<td>0.36</td>
<td>0.43</td>
<td>0.65</td>
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</tr>
<tr>
<td>Israel</td>
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<td>0.22</td>
<td>0.36</td>
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<tr>
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<td>0.07</td>
<td>0.11</td>
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<tr>
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<td>0.18</td>
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<tr>
<td>Qatar</td>
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<td>0.10</td>
<td>0.12</td>
<td>0.27</td>
<td>0.43</td>
<td>0.22</td>
<td>0.6</td>
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<tr>
<td>Saudi Arabia</td>
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<td>0.05</td>
<td>0.2</td>
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<td>0.18</td>
<td>0.32</td>
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<tr>
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<td>0.01</td>
<td>0.01</td>
<td>0.2</td>
<td>0.18</td>
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</table>

### Wavelet Correlation of India with American Countries

<table>
<thead>
<tr>
<th>Country</th>
<th>Pearson Correlation</th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>D4</th>
<th>D5</th>
<th>D6</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
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<td>0.28</td>
<td>0.38</td>
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<td>0.27</td>
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<td>0.25</td>
<td>0.43</td>
<td>0.65</td>
</tr>
<tr>
<td>Canada</td>
<td>0.34</td>
<td>0.32</td>
<td>0.28</td>
<td>0.35</td>
<td>0.45</td>
<td>0.53</td>
<td>0.58</td>
</tr>
</tbody>
</table>

**Note:** The wavelet scales are such that D1 = 2-4 weeks period, D2 = 4-8 weeks period, D3 = 8-16 weeks period, D4 = 16-32 weeks period, D5 = 32-64 weeks period, D6 = 64-128 weeks period.
Figure 1: Wavelet Cross correlation between stock returns of India and Japan using Daubechies least asymmetric (LA) wavelet filter of length 4.

Note: The red lines denote upper and lower bounds of the 95% confidence interval. The black line shows the cross correlation at different lags. Level 1=2~4 weeks, Level 2= 4~8 weeks, Level 3= 8~16 weeks, Level 4= 16~32 weeks, Level 5= 32~64 weeks and Level 6= 64~128 weeks. In order to preserve space, we did not report plots for other countries. However, the estimation results and plots are available on request.
Figure 2: Wavelet Coherence Plots (Power Spectrum) Between Indian & Global Equity markets
Note: Wavelet Coherence power spectrum plots: X’asis depicts the time (in observations or data points) and y’asis depicts wavelet scales. The red shade represents the correlation about 80%. While as blue shade represents correlation about 20% and the yellow shade illustrates correlation about 60%. The black bounded lines describes the confidence band at 95% level. Data points from 600-900 represents 2008-09 crises period time.